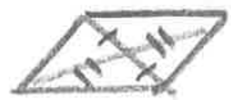
Unit 5 Study Guide

Name ~~AK~~ S____

1. Which information is needed to show that a parallelogram is a rectangle?

A. The diagonals bisect each other.
B. The diagonals are congruent.
C. The diagonals are congruent and perpendicular.
D. The diagonals bisect each other and are perpendicular.

2. Using A-D from #1, which information is needed to prove a parallelogram?



A) from # 1.
Diagonals bisect

3. Given the points $\boldsymbol{P(2,-1)}$ & $\boldsymbol{Q(-9,-6)}$, what are the coordinates of the point on the directed line segment $\overline{PQ}$ that partitions $\overline{PQ}$ into the ratio $\frac{3}{2}$?

$\frac{3}{5}$

A. $\left(-\frac{23}{5}, -4\right)$
B. $\left(-\frac{12}{5}, -3\right)$
C. $\left(\frac{5}{3}, \frac{8}{3}\right)$
D. $\left(-\frac{5}{3}, -\frac{8}{3}\right)$

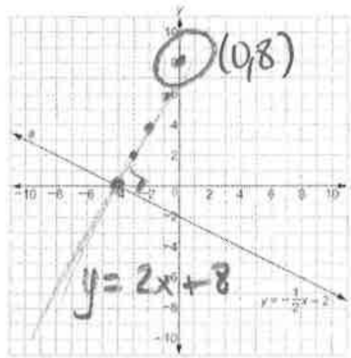
$x = 2 + \frac{3}{5}(-9-2) = 2 + \frac{3}{5}\left(\frac{-11}{1}\right)$

$x_2 - x_1$ $\quad \frac{5}{5}\cdot\frac{2}{1} + \frac{-33}{5} = \frac{10-33}{5}$

$y = -1 + \frac{3}{5}\left(\frac{-5}{1}\right) = \frac{5}{5}\cdot\frac{-1}{1} - \frac{15}{5} = \frac{-5-15}{5}$

Δy

4. An equation of a **line a** is $y = -\frac{1}{2}x - 2$. **See graph.**



What is the equation of the line that is perpendicular to line a shown on the graph and passes through point $\boldsymbol{(-4, 0)}$.

A. ~~$y = -\frac{1}{2}x + 2$~~
B. ~~$y = -\frac{1}{2}x + 8$~~
C. $y = 2x \cancel{-2}$
D. $y = 2x + 8$

$m = -\frac{1}{2} \rightarrow \frac{2}{1}$

$0 = \frac{2}{1}\left(\frac{-4}{1}\right) + b$

$0 = -8 + b$

$b = 8$

5. Which point is on a circle with a center of $(3, -9)$ and a radius of 5?

A. $(-6, 5)$
B. $(-1, 6)$
C. $(1, 6)$
D. $(6, -5)$

$(x-3)^2 + (y+9)^2 = 25$

$(6-3)^2 + (-5+9)^2 = 25$

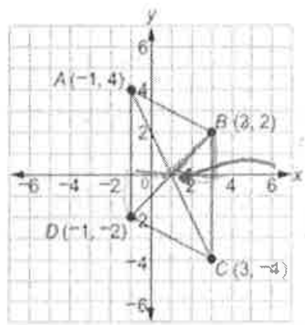
$3^2 + 4^2 = 25$

$9 + 16 = 25$

$25 = 25$

6. Parallelogram ABCD has vertices as shown.

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$\Delta x \quad \Delta y$



Write out the two sets, AC & BD, of the full distance formulas set equal to each other that would be used to prove that the diagonals of ABCD bisect each other? Then solve.

DB $\sqrt{(3-1)^2 + (2-0)^2} = \sqrt{(1-(-1))^2 + (0-2)^2}$

$\sqrt{2^2 + 2^2} = \sqrt{(2)^2 + (-2)^2}$ $\quad \sqrt{8} = \sqrt{8}$

AC $\sqrt{(1-(-1))^2 + (0-4)^2} = \sqrt{(3-1)^2 + (-4-0)^2}$

$\sqrt{2^2 + (-4)^2} = \sqrt{2^2 + (-4)^2}$ $\quad \sqrt{20} = \sqrt{20}$

Use the information provided to write the standard form of a circle.

7. $Center: (2\sqrt{3}, -5\sqrt{2}); Radius = \sqrt{13}$

$(x - 2\sqrt{3})^2 + (y + 5\sqrt{2})^2 = 13$

8. $Center$: (4, −14) and the point (6, 11) that lies on the circle.

$(x-4)^2 + (y+14)^2 = 629$

$r = \sqrt{(2)^2 + (25)^2} = \sqrt{629}$

$\Delta x \quad \Delta y$

Use the information provided to write the general conic form of a circle.

9. $(x + 10)^2 + (y - 7)^2 = 9$

$x^2 + 20x + 100 + y^2 - 14y + 49 - 9 = 0$

$x^2 + y^2 + 20x - 14y + 140 = 0$

10. $(x - 14)^2 + (y + 14)^2 = 9$

$x^2 - 28x + 196 + y^2 + 28y + 196 - 9 = 0$

$x^2 + y^2 - 28x + 28y + 383 = 0$

Use the information provided to write the standard form of a circle. Then idenify the center and radius length.

11. $x^2 + y^2 - 20x + 2y + 76 = 0$

$x^2 - 20x + \underline{100} + y^2 + 2y + \underline{1} = -76$

$\frac{-20}{2} = -10 \qquad \frac{2}{2} = 1 \qquad +100 \quad +1 \quad 25$

$(x-10)^2 + (y+1)^2 = 25$

Center: (10, -1); r = 5

12. $\frac{2x^2 + 2y^2 + 28x + 24y + 21 = 0}{2}$

$x^2 + 28x + \underline{49} \; y^2 + 12y + \underline{36} = -10.5$

$-\frac{14}{2} = (7)^2 \qquad \frac{12}{2} = (6)^2 \qquad +49 \quad +36$

$(x+7)^2 + (y+6)^2 = 74.5$

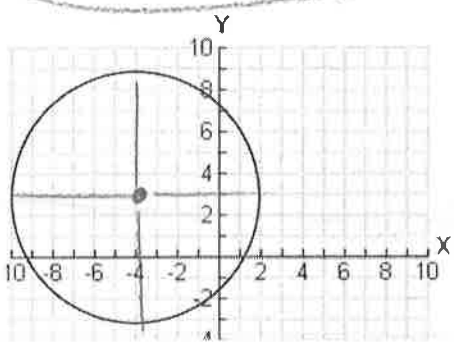
Center: (-7, -6); $r = \sqrt{74.5}$

Find the center and the radius length to write the standard form of each circle.

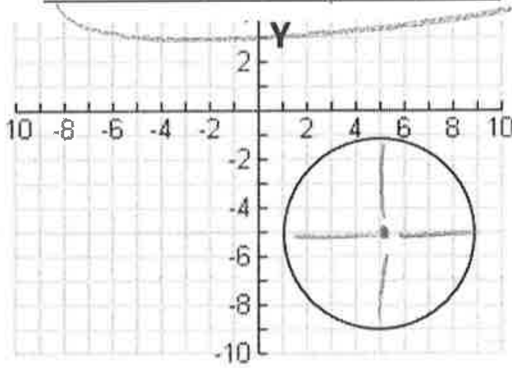
13. $(x+4)^2 + (y-3)^2 = 36$

c: (-4, 3)

r = 6



14. $(x-5)^2 + (y+5)^2 = 16$

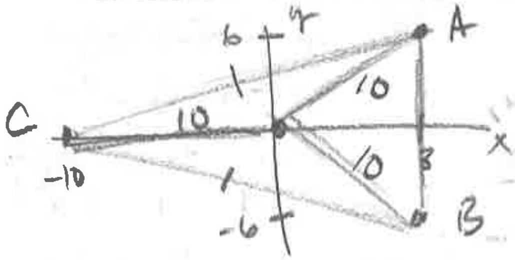


c: (5, -5)

r = 4

(0,0)

15. Prove or disprove that the points $A(8,6), B(8,-6)$ *and* $C(-10,0)$ are the vertices of an isosceles triangle inscribed in the circle centered at the origin Q and passing through the point $P(\cancel{0}3, \sqrt{91})$.



$d_{\overline{AC}} = \sqrt{(18)^2 + (6)^2}$ ✓

$d_{\overline{CB}} = \sqrt{(18)^2 + (-6)^2}$

$\triangle ABC$ is isosceles.

$r_P = \sqrt{(3-0)^2 + (\sqrt{91}-0)^2} = \sqrt{100} = 10$

$r_A = \sqrt{(8-0)^2 + (6-0)^2} = \sqrt{100} = 10$

$r_B = \sqrt{(8-0)^2 + (-6-0)^2} = \sqrt{100} = 10$

$r_C = 10$ All radius = 10

On a coordinate plane, a local television station ~~in sites~~ is located at the origin and has a broadcast range of 50 miles.

16) Write an equation that represents the region covered by this television station.

$x^2 + y^2 = 2{,}500$

ctr @ origin

17) Can a person who lives 18 miles to the East and 35 miles North of the station watch this TV station?

$d = \sqrt{(18)^2 + (35)^2}$

$= \sqrt{324 + 1225}$

$= 39.36$ miles

yes $39.36 < 50$ so he can watch the station

You're a city planner, so you know that streets run north to south and avenues run east to west. Your friend Melissa lives at the corner of 3rd Street and 28th Avenue. Her sister Rebecca lives at the corner of 27th Street and 16th Avenue. If necessary, draw a graph to find the cross street that:

18. Is halfway between their homes.

Ave St

$\text{Midpt} = \left(\frac{28+16}{2}, \frac{3+27}{2}\right)$

$= \left(\frac{44}{2}, \frac{30}{2}\right)$

The cross street is @ 22nd Ave & 15 st.

19. Is $\frac{2}{3}$ of the way from Melissa's to Rebecca's.

$\text{Ave} = 28 + \frac{2}{3}\left(\frac{-12}{1}\right) = 28 - 8 = 20$

$\text{St} = 3 + \frac{2}{3}\left(\frac{24}{1}\right) = 3 + 16 = 19$

20th Ave & 19th street

20. Separates their homes in a ratio of 3: 1.

A B $\frac{A}{A+B}$

$\text{Ave} = 28 + \frac{3}{4}\left(\frac{-12}{1}\right) = 28 - 9 = 19$

$\text{st} = 3 + \frac{3}{4}\left(\frac{24}{1}\right) = 3 + 18 = 21$

19th AVE & 21st street

21. Separates their homes in a ratio of $\frac{1}{5}$.

$\text{Ave} = 28 + \frac{1}{6}\left(-\frac{12}{1}\right) = 28 - 2 = 26$

$\text{St} = 3 + \frac{1}{6}\left(\frac{24}{1}\right) = 3 + 4 = 7$

26th AVE & 7th street

Determine if point A lies on a circle with center C and point P which is known to lie on the circle.

22. $A(5,0), C(0,0), P(3,4)$

ctr ptonC

d_{CP} or $r = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$ ✓

d_{CA} possible $r = \sqrt{(5)^2 + (0)^2} = \sqrt{25} = 5$ ✓

yes, pt A lies on circle C.

23. $A(0,4), C(2,1), P(5,3)$

ctr ptonC

$d_{CP} = \sqrt{(3)^2 + (2)^2} = \sqrt{13}$ ✓

d_{CA} possible $r = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$ ✓

yes, pt A lies on circle C.

For each figure using, prove the type of quadrilateral, using distance and, or slope. Keep diagonals in mind.

24. $ABCD$: $A(1,2), B(2,5), C(4,3), D(5,6)$

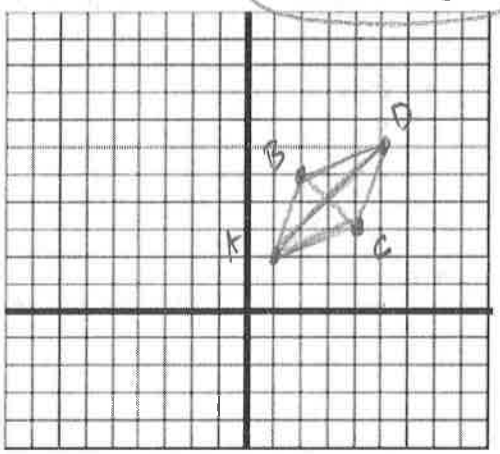
If parallelogram, diagonals BC & AD bisect one another

slope

$m_{BD} = 1/3$, $m_{AC} = 1/3$ } parallel & components will result in the same distances

$m_{AB} = 3/1$, $m_{CD} = 3/1$ } parallel

rhombus



25. $EFGH$: $E(4,1), F(-2,3), G(2,-5), H(-4,-3)$

If SQ, all sides ≅, opp. sides parallel.

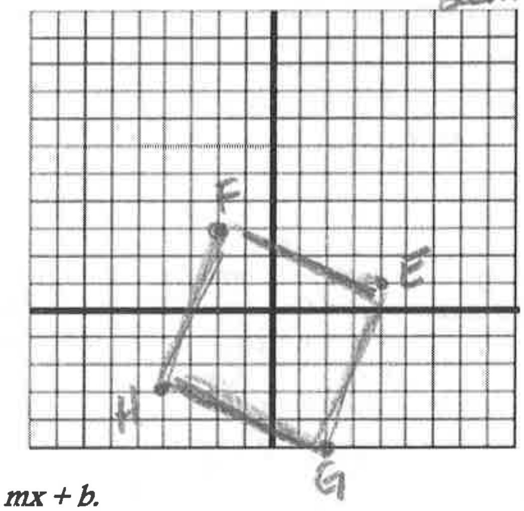
SQUARE

slope

$m_{FE} = -2/6$, $m_{HG} = -2/6$ } parallel

$m_{GF} = 6/2$, $m_{HF} = 2/6$ } parallel

these components will all have the same distance $\sqrt{40}$ or $2\sqrt{10}$.



Write the equation of the lines below in slope-intercept form: $y = mx + b$.

26. Through $(-4, 5)$ and parallel to $y = -\frac{3}{2}x - 5$.

$5 = -\frac{3}{2}\left(\frac{-4}{1}\right) + b$

$5 = \frac{12}{2} + b$

$5 = 6 + b$

$b = -1$

$\boxed{y = -\frac{3}{2}x - 1}$

27. Through $(4, 1)$ and perpendicular to $y = -2x - 2$

new $m = 1/2$

$1 = \frac{1}{2}(4) + b$

$1 = 2 + b$

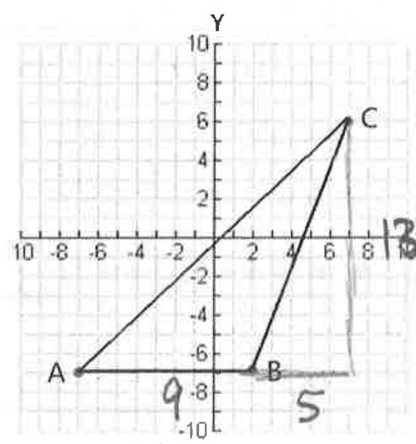
$b = -1$

$\boxed{y = \frac{1}{2}x - 1}$

Find the are and perimater of the following triangle. Simplest form required. Reminder: Draw altitude to find height.

28. Area = $58.5\,u^2$

$\frac{b \cdot h}{2} = \frac{9 \cdot 13}{2} = \frac{117}{2} = 58.5$



13 ht of ABC

AB = 9

29. Perimeter = $9 + \sqrt{194} + \sqrt{365}$ or 42.03 units

$d_{BC} = \sqrt{(5)^2 + (13)^2} = \sqrt{194}$

$d_{AC} = \sqrt{(14)^2 + (13)^2} = \sqrt{365}$