LESSON 33

Conditional Probability

UNDERSTAND A **conditional probability** is the probability that an event will occur, given that one or more events have already occurred. You can write the conditional probability of event A happening assuming that event B has already occurred as $P(A \mid B)$. This is read as the probability of "A given B." The conditional probability $P(A \mid B)$ is equal to the joint probability for A and B divided by the marginal probability of B:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

UNDERSTAND Conditional probability is not the same thing as compound probability. To find the probability of compound events, the probabilities of individual events are determined and then used to find the sample space for two or more of the events happening together. In the case of conditional probability, the sample space of the experiment is often reduced by additional information.

For example, suppose you toss two coins—two independent events. The sample space includes four possible outcomes: {HH, HT, TH, TT}. The probability that both coins will land on heads can be found by multiplying the probability of the first toss landing on heads by the probability of the second toss landing on heads.

$$P\text{ (heads} \cap \text{heads)} = P\text{(heads first)} \cdot P\text{(heads second)} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Suppose you already tossed the first coin and you already know that it landed on heads. The probability of both coins landing on heads is now different. The sample space is smaller because {TH, TT} are no longer possible outcomes. Given that the first coin toss resulted in heads, there are only two possible outcomes now, {HH, HT}. So, the probability of getting {HH} is now $\frac{1}{2}$. This can be shown using the conditional probability formula:

$$P\text{(heads second} \mid \text{heads first)} = \frac{P\text{(heads second} \cap \text{heads first)}}{P\text{(heads first)}} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Notice that the probability of P(heads second | heads first) is the same as P(heads second). This is because if events A and B are independent, then $P(A \text{ given } B) = P(A)$ and $P(B \text{ given } A) = P(B)$. Recall that for independent events, $P(A \cap B) = P(B \cap A) = P(A)P(B)$.

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$		$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$
$P(A \mid B) = \frac{P(A)P(B)}{P(B)}$	for independent events	$P(B \mid A) = \frac{P(B)P(A)}{P(A)}$
$P(A \mid B) = P(A)$	for independent events	$P(B \mid A) = P(B)$

Practice

Use the information below for questions 1 and 2.

A white number cube and a red number cube, each with faces numbered 1 to 6, are tossed at the same time.

1. What is the probability of both cubes landing on 4 if you know that the white cube landed on 4?

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2. What is the probability of both cubes landing on even numbers if you know that the white cube landed on 4?

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REMEMBER $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Use the information below for questions 3 and 4.

A quarter, a nickel, and a penny are all tossed at the same time.

3. What is the probability of all 3 coins landing on heads if you know that the quarter and the nickel landed on heads?

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4. What is the probability of all 3 coins landing on heads if all you know is that the quarter landed on heads?

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HINT Write the sample space and consider how the conditions change it.

Write *true* or *false* for each statement. If the statement is false, rewrite it so it is true.

5. A conditional probability is the probability that an event will occur, given that one or more events may occur in the future.

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6. Conditional probability and compound probability are not the same thing.

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7. A and B are independent events if $P(A \mid B) = P(B)$ and $P(B \mid A) = P(A)$.

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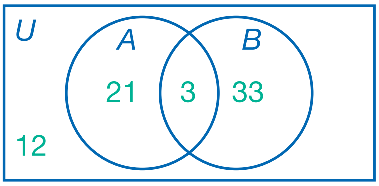
Choose the best answer.

8. The Venn diagram shows the number of possible outcomes in sets *A* and *B* and their intersection. What is $P(B \mid A)$ for this Venn diagram?



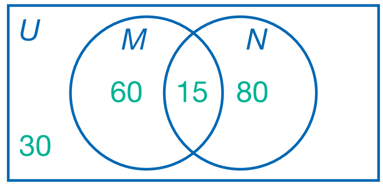
A. $\frac{1}{12}$

B. $\frac{1}{11}$

C. $\frac{1}{8}$

D. $\frac{1}{7}$

9. The Venn diagram shows the number of possible outcomes in sets *M* and *N* and their intersection. What is $P(N \mid M)$ for this Venn diagram?



A. 0.1875

B. 0.2

C. 0.25

D. 0.5

10. The two-way frequency table shows the quiz scores of students in the same biology class and whether or not each student studied for the quiz.

	Score ≥ 90	Score < 90
Studied	12	3
Did Not Study	1	14

What is the probability that a randomly selected student from the class had a score less than 90, given that the student did not study for the quiz?

A. $\frac{7}{15}$

B. $\frac{17}{30}$

C. $\frac{4}{5}$

D. $\frac{14}{15}$

11. The two-way frequency table shows the number of boys and girls who are working on the school play and whether they are performers or stage crew members.

	Performer	Stage Crew
Boys	14	18
Girls	21	7

What is the chance that a randomly selected student working on the school play is a boy, given that the student is a member of the stage crew?

A. 30%

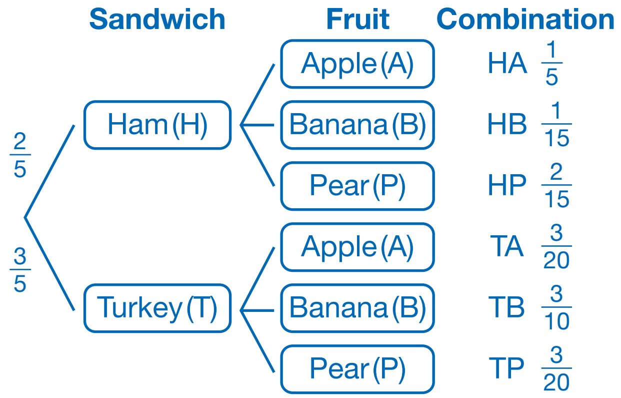
B. 56.25%

C. 72%

D. 78.26%

Use the information and tree diagram below for questions 12 and 13.

There were 50 boxed lunches prepared for a school meeting. Each boxed lunch contains either a ham sandwich or a turkey sandwich and either an apple, a banana, or a pear. Alice is the first to choose a boxed lunch. She chooses a box marked "ham sandwich," but she does not pay attention to which type of fruit is inside.



12. Given that she chose a ham sandwich, what is the probability that she chose a boxed lunch

with an apple? ______________________________

with a banana? ______________________________

with a pear? ______________________________

13. Compare $P(\text{A}|\text{T})$ to $P(\text{P}|\text{T})$.

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Choose the best answer.

14. Event A is independent of event B if which of the following is true?

A. $P(A \mid B) = P(A)$

B. $P(A \mid B) = P(B)$

C. $P(A \mid B) = P(A)P(B)$

D. $P(A \mid B) = \frac{P(A \cap B)}{P(A)}$

15. If $P(A) = 0.3$ and $P(B) = 0.2$ and A and B are independent events, which of the following must be true of $P(B \mid A)$?

A. $P(B \mid A) = 0.3$

B. $P(B \mid A) = 0.2$

C. $P(B \mid A) = 0.15$

D. $P(B \mid A) = 0.06$

Use the two-way frequency table and information below for questions 16–19.

A poll asked voters of different ages if they were for or against a proposal to build a new public library.

Ages	For	Against	No Opinion	Total
18–39	30	42	3	75
40–59	80	55	5	140
60 and over	120	13	2	135
Total	230	110	10	350

16. What is the chance that a voter has no opinion on the proposal, given that the voter is younger than 40 years old?

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17. What is the probability that a voter is for the proposal, given that he or she is 60 or over?

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18. What is the probability that a voter who is against the proposal is age 40–59?

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19. Is a voter's opinion about the proposed new library independent of age? How do you know?

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A total of 60 students in grades 10, 11, and 12 were surveyed and asked whether or not they play on a sports team. Of those surveyed, 20 out of 30 tenth-grade students play on a team, 8 out of 12 eleventh-grade students play on a team, and 12 out of 18 twelfth-grade students play on a team.

20. CREATE Enter the above information in the two-way frequency table, List the joint and marginal frequencies.

	Play on Team(s)	No Team	Total
10th Grade			
11th Grade			
12th Grade			
Total			

21. JUSTIFY Is participation in team sports independent of grade level? Justify your answer.

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