LESSON 32 Compound Probability

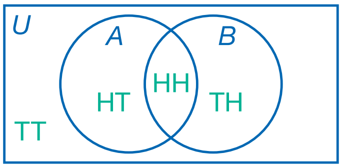
Independent and Dependent Events

UNDERSTAND Sometimes, two or more events happen together. This is called a **compound event.** There are two types of compound events.

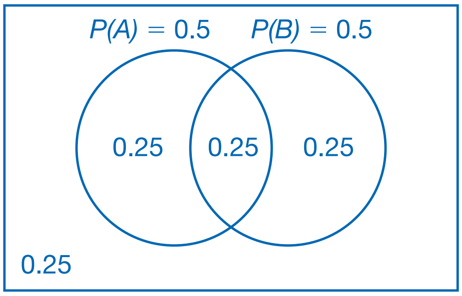
Independent events are events in which the outcome of one event does not affect the outcome of other events. For example the outcome of tossing one coin does not affect the outcome of a second coin toss.

With **dependent events**, the outcome of one event affects the outcome of other events. An example is picking a marble from a bag containing marbles of different colors and then picking a second marble from the bag without replacing the first one. Whatever color was picked first affects the probabilities of the second pick.

UNDERSTAND The probability that two events A and B will occur at the same time can be represented as the intersection of sets A and B. Suppose two fair coins are tossed. Let's say that event A is the first coin landing on heads, and event B is the second coin landing on heads. This Venn diagram represents the possible outcomes and shows that $A \cap B = \{HH\}$.



A second Venn diagram can be created to represent the probabilities associated with tossing these two coins, as shown.



Multiplication Rule for Independent Events: The probability of the intersection of two subsets composed of independent events is equal to the product of their probabilities:

$P(A \cap B) = P(A)P(B)$

Substitute the probabilities into the formula to show that events A and B are independent.

$$P(A \cap B) = P(A)P(B)$$

$$0.25 \stackrel{?}{=} (0.5)(0.5)$$

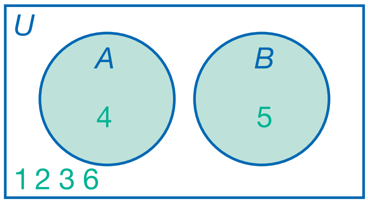
$$0.25 = 0.25 \checkmark$$

If two events are independent, you can multiply their probabilities to find the probability that both will occur. This formula can also be used to determine if events are independent. If the probability of the events happening together or one after the other is equal to the product of their probabilities, the events are independent.

The Addition Rule

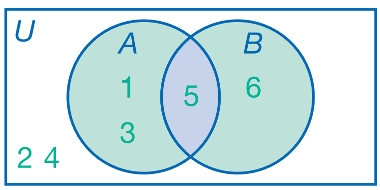
UNDERSTAND If two events cannot happen at the same time, they are **mutually exclusive events.** The probability of either of two mutually exclusive events occurring is equal to the sum of their individual probabilities. For example, suppose you toss a standard number cube. You cannot toss a 4 and a 5 at the same time, as shown by the Venn diagram. They are mutually exclusive events. So, the probability of tossing a 4 or a 5 is:

$$P(4 \text{ or } 5) = P(4) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$



Events that are not mutually exclusive may have outcomes in common. For example, suppose you toss a number cube. Consider the event of a toss resulting in an odd number and the event of a toss resulting in a number greater than 4. Those events overlap because 5 is an element of both sets, as shown by the Venn diagram below. So, to find the probability of tossing an odd number or a number greater than 4, apply the Addition Rule.

> **Addition Rule:** The probability of the union of two subsets is equal to the sum of their individual probabilities minus the probability of the intersection of the subsets.
>
> $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$P(\text{odd or } >4) = P(\text{odd}) + P(>4) - P(\text{odd and } >4) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Consider the events as sums of the probabilities of mutually exclusive events.

$P(\text{odd}) = P(1 \text{ or } 3 \text{ or } 5) = P(1) + P(3) + P(5)$ $\quad P(>4) = P(5 \text{ or } 6) = P(5) + P(6)$

$P(\text{odd and } >4) = P(5)$

$P(\text{odd}) + P(>4) - P(\text{odd and } >4) = P(1) + P(3) + P(5) + \cancel{P(5)} + P(6) - \cancel{P(5)}$

$P(\text{odd}) + P(>4) - P(\text{odd and } >4) = P(1) + P(3) + P(5) + P(6) = P(\text{odd or } >4)$

Practice

Write an appropriate word or phrase in each blank.

1. ________________ events are events in which the outcome of one event does not affect the outcome of the other event or events.

2. The probability of the intersection of two subsets A and B that are composed of independent events is equal to the ______ of their probabilities.

3. The ______ Rule states that the union of two dependent events A and B is equal to the sum of $P(A)$ and $P(B)$ minus $P(A \cap B)$.

Events *A* and *B* (and *C*, if given) are independent events. Their probabilities are given below. Find the probability asked for.

4. $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$P(A \cap B) =$ ______

5. $P(A) = 0.7$, $P(B) = 0.3$

$P(A \cap B) =$ ______

6. $P(A) = 0.3$, $P(B) = 0.2$

$P(A \cap B) =$ ______

7. $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{2}$

$P(A \cap B \cap C) =$ ______

HINT $P(A \cap B \cap C) = P(A \cap B)P(C)$

Events *A* and *B* are not mutually exclusive. Find the probability of their union.

8. $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$

$P(A \cup B) =$ ______

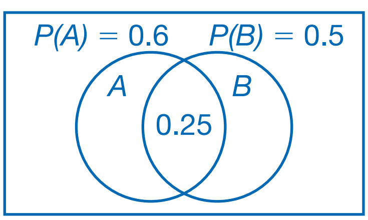
9. $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$

$P(A \cup B) =$ ______

REMEMBER $\cup$ is the union of the sets, and $\cap$ is the intersection of the sets.

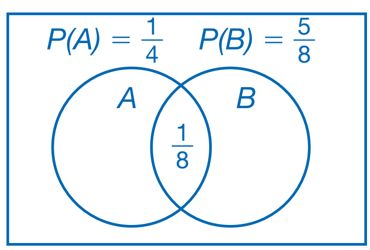
Find $P(A \cup B)$ given the probabilities in the Venn diagrams.

10.



$P(A \cup B) =$ ______

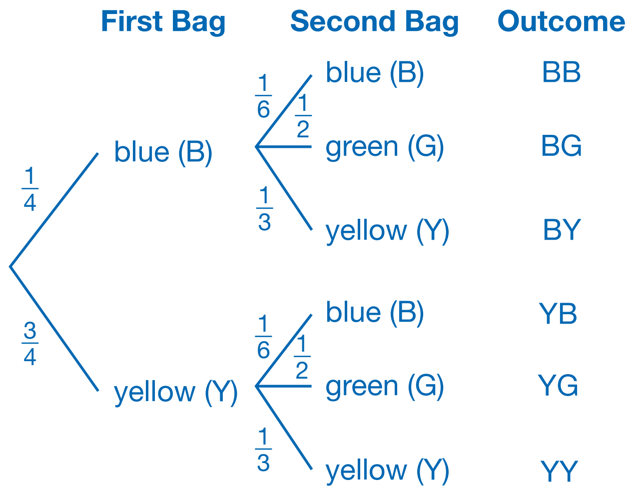
11.



$P(A \cup B) =$ ______

Use the tree diagram and the information below for questions 12–16.

Mei-lin has two bags with colored tiles in them. She will select one tile from the first bag and then select one tile from the second bag, recording their colors. The tree diagram shows the probability for each possible outcome.



12. What is the sample space for the experiment? ______________________________

13. Are all of the outcomes equally likely? How do you know?

____

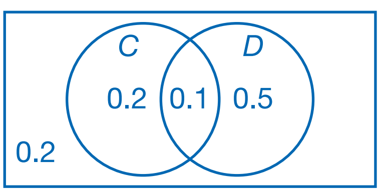
14. What is the probability of selecting a blue tile first and a green tile second? ________

15. What is the probability of selecting two yellow tiles? ________

16. One outcome above has a probability of $\frac{1}{12}$. Which outcome is it? ________

Determine whether or not events *C* and *D* are independent events.

17.

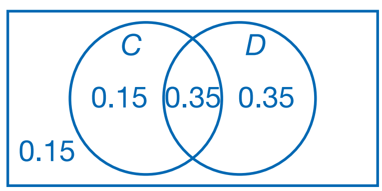


$P(C) =$ ________

$P(D) =$ ________

Event C and *D* ____________ independent.

18.



$P(C) =$ ________

$P(D) =$ ________

Event *C* and *D* ____________ independent.

Use the two-way frequency table and information below for questions 19 and 20.

Girls and boys at Rock Ridge Nature Camp had to sign up for either rock climbing or archery. The table shows which activity the boys and girls at the camp chose.

	Rock Climbing (R)	Archery (A)
Boys (B)	34	16
Girls (G)	23	27

19. Find $P(R \cup B)$. What does this probability represent?

20. Find $P(G \cup A)$. What does this probability represent?

Use the two-way frequency table and information below for questions 21 and 22.

The two-way frequency table shows the results of a survey of 11th-grade and 12th-grade students at Hamilton High School. Students were asked if they do community service.

	Does Community Service (C)	No Community Service (N)	Total
11th Grade (E)	24	12	36
12th Grade (T)	16	8	24
Total	40	20	60

21. Suppose an 11th-grade or 12th-grade student at Hamilton High School is selected at random.

What is the probability that the student is either in 11th grade or does community service (or both)?

What is the probability that the student is either in 12th grade or does community service (or both)?

22. SHOW Are the events "does community service" and "11th grade" independent? Explain your reasoning.

23. CREATE Shelley will spin three spinners. The first spinner has two congruent sectors: one red and one blue. The second spinner has four congruent sectors: one red, two blue, and one yellow. The third spinner has nine congruent sectors, three of which are red, and the rest of which are blue. Create a tree diagram to represent all the possible outcomes for this experiment. Label the probabilities on each branch. Then determine the probability of spinning blue on all three spinners.