



Using Venn Diagrams to Find Probabilities

UNDERSTAND The probability of an event, A , occurring is represented as $P(A)$. Probability is expressed as a number from 0 to 1 that shows how likely the event is to occur. It can be written as a fraction, a decimal, or a percent and is given by the following ratio:

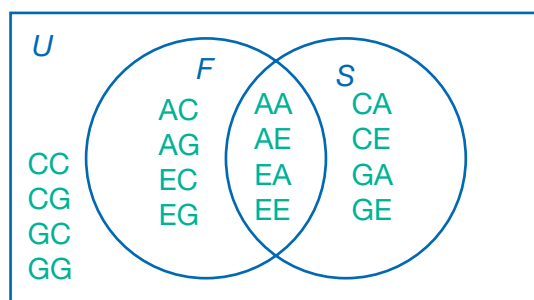
$$P(A) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

So, for example, if you toss a number cube, there are 6 possible outcomes: {1, 2, 3, 4, 5, 6}. Suppose you want to know the probability of tossing an even number. In that case, there are 3 favorable outcomes: {2, 4, 6}.

$$P(\text{even}) = \frac{3}{6} = \frac{1}{2}$$

This probability can be written as $\frac{1}{2}$, 0.5, or 50%.

UNDERSTAND **Joint probability** is the probability that two events will occur at the same time or one after the other. For example, suppose you have two bags. One contains four cards lettered A, C, E, and G. The other also contains four cards also lettered A, C, E, and G. Suppose you want to determine the probability of selecting vowels (A or E) from both bags. Placing the outcomes in a Venn diagram allows you to analyze them. Circle F below represents selecting a vowel from the first bag. Circle S represents selecting a vowel from the second bag.



The Venn diagram shows that there are 16 possible outcomes. Of those, 8 involve selecting a vowel from the first bag, 8 involve selecting a vowel from the second bag, 4 involve selecting two vowels, and 4 involve selecting no vowels. Note that each outcome is equally likely. The number of outcomes in a region divided by the total number of outcomes shown by the diagram gives the probability for the event that represents that region.

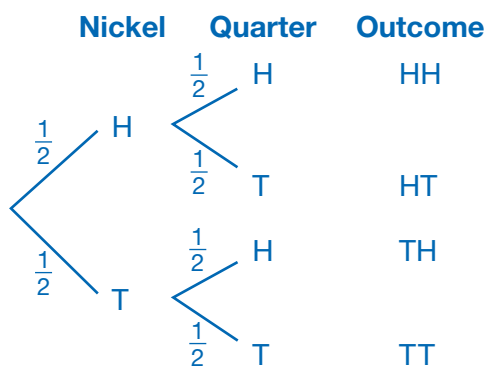
There are 4 outcomes in the intersection of the sets, so the probability that you will select a vowel from both bags is $\frac{4}{16} = \frac{1}{4}$.

Tree Diagrams and Two-Way Tables

UNDERSTAND Suppose you toss two fair coins—a nickel and a quarter—at the same time. Each coin can land on either heads (H) or tails (T). So for each individual coin toss, the probabilities are:

$$P(H) = \frac{1}{2} \quad P(T) = \frac{1}{2}$$

However, the probability of both coins landing on heads, $P(HH)$, is not $\frac{1}{2}$. Drawing a **tree diagram** to represent all the possible outcomes can help you see this. The last column of the tree diagram below shows that there are 4 possible outcomes: {HH, HT, TH, TT}. Each outcome is equally likely, so the probability of each outcome is $\frac{1}{4}$. That means $P(HH) = \frac{1}{4}$.



UNDERSTAND Probabilities can also help you understand data that are collected by surveying a representative **sample** of people drawn from a larger **population**. If you want to compare two categorical variables such as gender and reading habits, you can construct a **two-way frequency table** like the one below. The table shows **joint frequencies** and **marginal frequencies**.

	Reads for Pleasure	Reads Only for School	Total
Boys	20	30	50
Girls	30	20	50
Total	50	50	100

Joint frequencies are in the body of the table.

Marginal frequencies are in the “Total” row and “Total” column.

If you divide a particular frequency by one of the totals, you can determine the **relative frequency** for a row, a column, or the entire table. For example, $\frac{20}{50}$, or 40%, of all the boys surveyed said they like to read for pleasure, and $\frac{20}{100}$, or 20%, of all the people surveyed above were boys who said they like to read for pleasure. This means that if you select a boy from the population at random, there would be a 40% chance that he likes to read; and if you select a person from the population at random, there would be a 20% chance that person would be a boy who likes to read.

Practice

Identify the probability for each simple event.

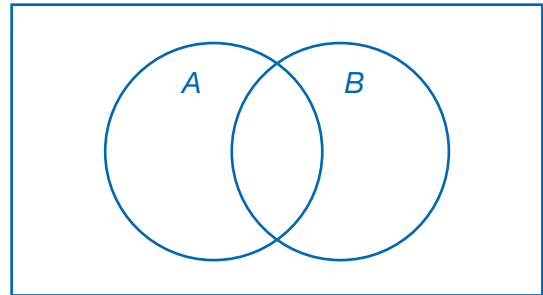
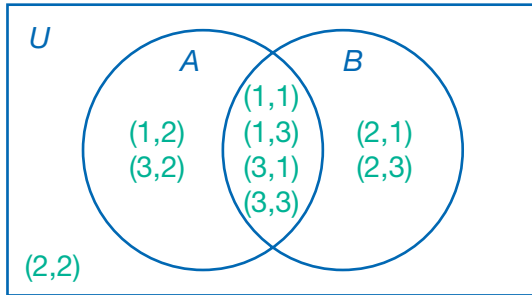
1. tossing a number cube with faces numbered 1 to 6 and getting 3

2. drawing a heart from a standard 52-card deck



For the Venn diagram and situation, determine $P(A - B)$, $P(B - A)$, $P(A \cap B)$ and $P(\overline{A \cup B})$. In the blank Venn diagram, write those probabilities in the appropriate sections.

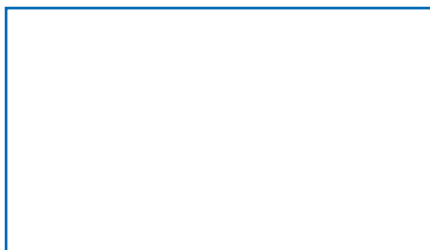
3. Two spinners each have sections numbered 1–3. Event A represents getting an odd number on the first spinner. Event B represents getting an odd number on the second spinner.



Use the information below for questions 4 and 5.

A bag contains 1 marble with red and black stripes, 1 marble with blue and gray stripes, 1 marble with red and green stripes, 1 marble with blue and white stripes, 1 marble that is solid white, and 1 marble that is solid green. Curtis will reach into the bag and pick a marble without looking. Event R is picking a marble that has red stripes, and event B is picking a marble that has blue stripes.

4. Create a Venn diagram to show sets R and B



5. Determine each probability

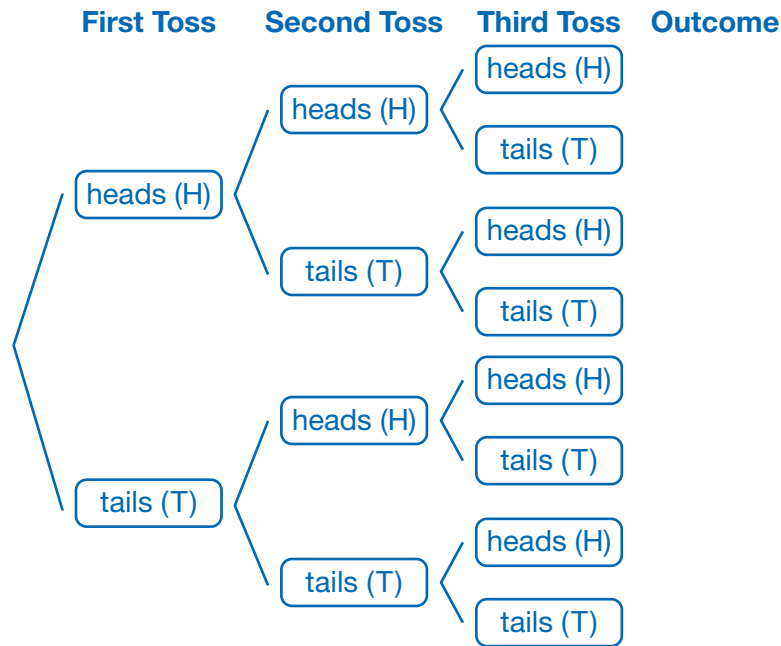
$P(R) =$ _____

$P(B) =$ _____

$P(R \cap B) =$ _____

$P(\overline{R \cup B}) =$ _____

The tree diagram represents tossing a fair coin 3 times. Use the tree diagram for questions 6–8.



- List all the possible outcomes on the tree diagram.
- Each outcome is equally likely. What is the probability of tossing 3 tails? _____
- What is the probability of tossing heads on at least 2 tosses? _____

Use this information for questions 9 and 10.

Fifty students were asked which television genre they like best. Five boys said drama, eight boys said sitcom, and twelve boys said reality. Eight girls said drama, ten girls said sitcom, and seven girls said reality.

- CREATE** Use the grid below to create a two-way frequency table for these data.

				Total
Boys				
Girls				
Total				

- EXPLAIN** What is the probability that a student who likes sitcoms best is a girl? Explain how you determined your answer.
