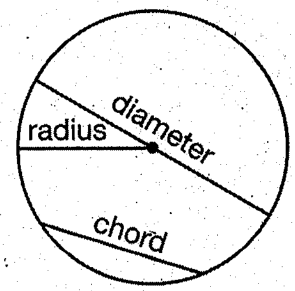
LESSON 16 Circles and Line Segments

Chords, Radii, and Diameters

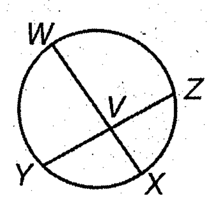
UNDERSTAND A **circle** is the collection of points that are equidistant from a given point, which is called the center. The distance from the center to a point on the circle is the length of a **radius** of the circle. So, every radius that can be drawn in the circle has the same length. A circle is usually named by its center, so a circle whose center is point Q is named circle Q, or $\odot Q$.

A **chord** is a line segment that has both endpoints on a circle. Two chords within the same circle can have different lengths. A **diameter** is a chord that passes through the center of a circle. Diameters are the longest possible chords in a circle, so all of a circle's diameters are the same length. The length of the diameter of a circle is twice the length of its radius.



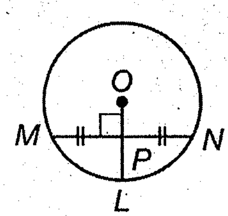
UNDERSTAND Relationships among these line segments can help you determine their lengths.

If two chords intersect and divide each other into segments, the product of the segments of one chord equals the product of the segments of the other chord.



$$WV \cdot VX = YV \cdot VZ$$

If a radius or a diameter intersects a chord and is perpendicular to the chord, it bisects the chord.

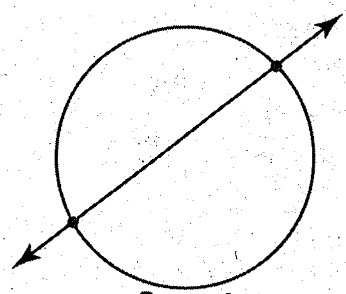


In circle O, $\overline{OL} \perp \overline{MN}$ and $MP = PN$.

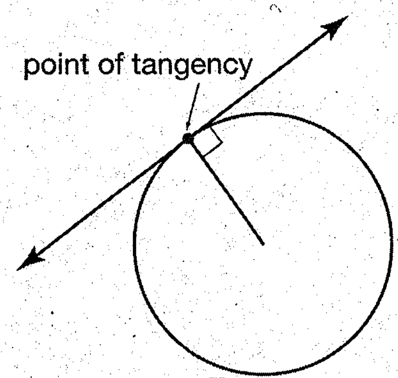
The converse is also true. If a radius or diameter bisects a chord, then it is perpendicular to the chord.

Secant Lines and Tangent Lines

UNDERSTAND A **secant line** is a line in the plane that intersects a circle at two points. A secant could be formed by extending the ends of a chord. A **tangent line** intersects a circle at exactly one point. That point is called the point of tangency. The radius drawn to a point of tangency is perpendicular to the tangent line through that point.

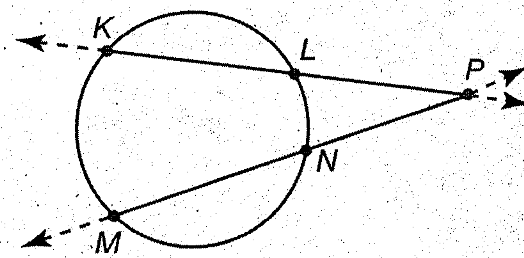


Secant



Tangent

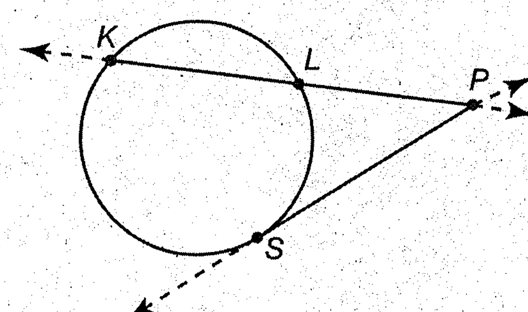
UNDERSTAND $\overleftrightarrow{KL}$ and $\overleftrightarrow{MN}$ are secant lines that intersect at point P. Segments such as KP and MP are called secant segments. The portion of a secant segment outside the circle, such as $\overline{LP}$ or $\overline{NP}$, is called an external segment. If two secant lines intersect outside a circle, the product of the secant segment and external segment of one secant line is equal to the product of the secant segment and external segment of the other secant line.



$KP \cdot LP = MP \cdot NP$

Imagine pulling secant $\overleftrightarrow{MN}$ down. Points M and N, which are located where the line intersects the circle, would move down the circle and come closer together. If you pulled the line down far enough, M and N would become the same point. The line would now be tangent to the circle.

If we call this single point of tangency S, the diagram is now changed so that secant $\overleftrightarrow{MN}$ is replaced by tangent $\overleftrightarrow{SP}$. Segment MP is replaced by segment SP, and segment NP is also replaced by segment SP. So, the product of KP and LP is equal to SP squared.



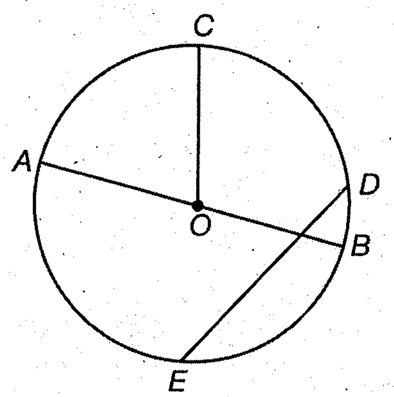
$KP \cdot LP = SP \cdot SP$

$KP \cdot LP = SP^2$

Practice

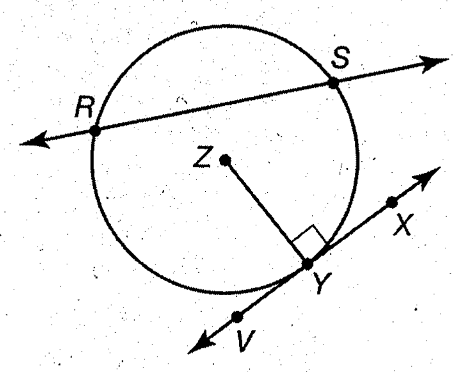
Give examples of the following in the given circles.

Name the following in Circle *O*:



1. The center: ______
2. A diameter: ______
3. Three radii: ____________
4. Two chords: ____________

Name the following in Circle *Z*:



5. A radius: ______
6. A chord: ______
7. A secant line: ______
8. A tangent line: ______

REMEMBER A secant line contains a chord.

Write *true* or *false* for each statement. If the statement is false, rewrite it so that it is true.

9. Every chord of a circle is also a diameter of the circle.

____

10. When two chords intersect inside a circle, the product of the divided segments of one chord equals the product of the divided segments of the other chord.

____

____

11. A diameter that intersects a chord bisects the chord.

____

12. A radius drawn to a point of tangency is perpendicular to the tangent line through that point.

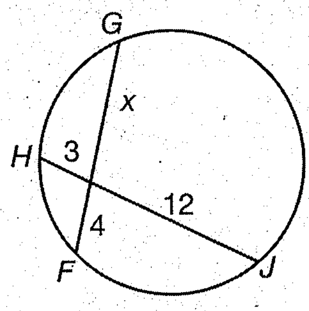
____

13. A secant line intersects a circle in exactly one point.

____

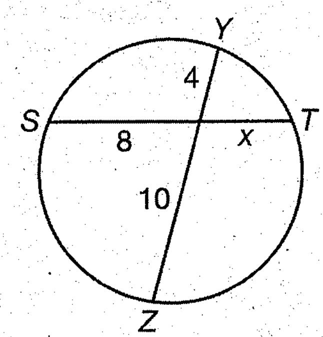
Each circle shows intersecting chords. Find the length represented by x in each circle.

14.



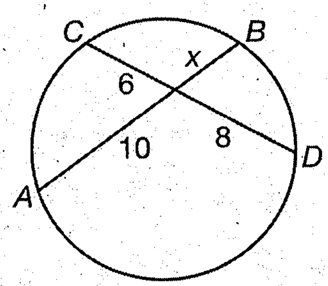
$x =$ ____________

15.



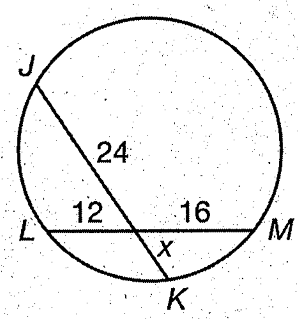
$x =$ ____________

16.



$x =$ ____________

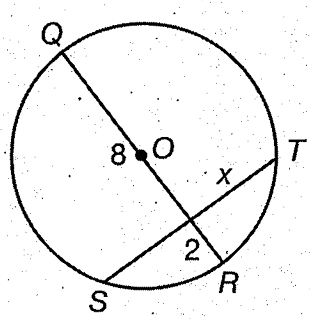
17.



$x =$ ____________

18. Point O is the center of this circle.

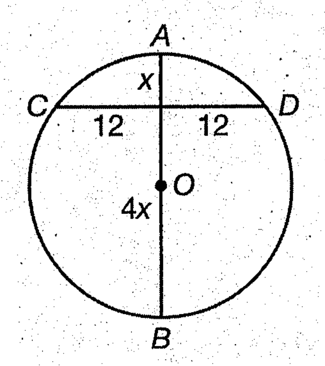
$QR = 10$



$x =$ ____________

19. Point O is the center of this circle.

$AB = 5x$

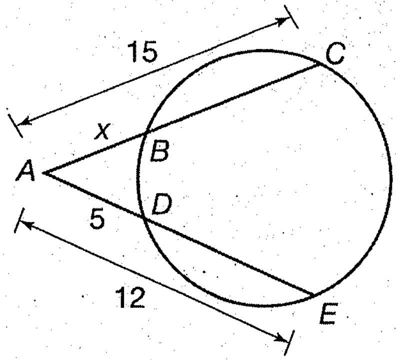


$x =$ ____________

Duplicating this page is prohibited by law. © 2016 Triumph Learning, LLC

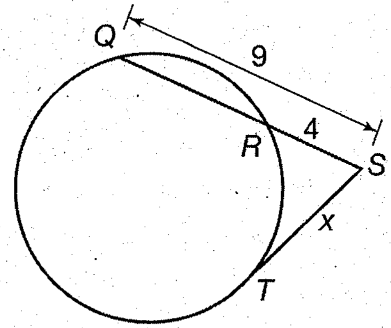
Find the length represented by *x* in each diagram.

20.



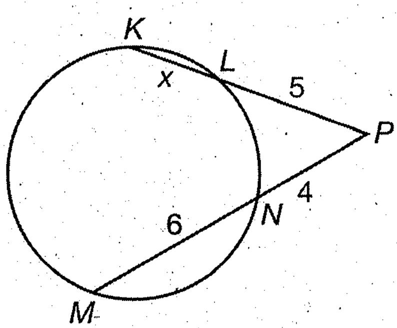
$x =$ ____________

21.



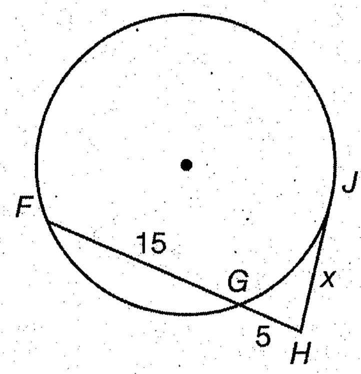
$x =$ ____________

22.



$x =$ ____________

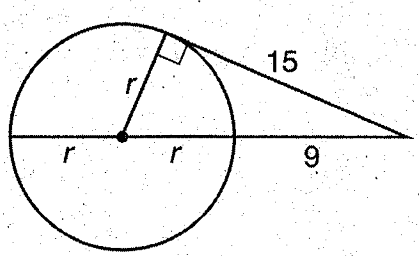
23.



$x =$ ____________

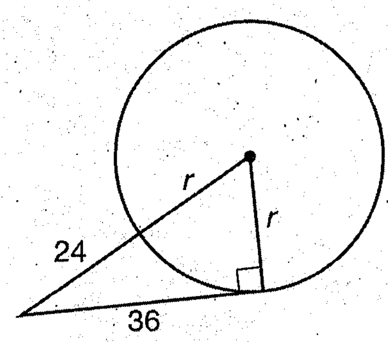
Find the length of the radius, *r*, for each circle.

24.



$r =$ ____________

25.



$r =$ ____________

Choose the best answer.

26. A diameter of a circle is perpendicular to a chord whose length is 14 centimeters. If the length of the shorter segment of the diameter is 5 centimeters, what is the length of its longer segment?

A. 2.8 cm **C.** 9.8 cm

B. 9 cm **D.** 19 cm

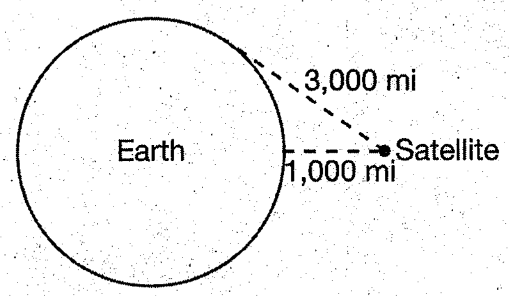
27. The diameter of a circle is 7 meters. If the diameter is extended 2 meters beyond the circle to point *P*, how long is a tangent segment from point *P* to the circle? Give your answer to the nearest tenth of a meter.

A. 3.7 m **C.** 9.0 m

B. 4.2 m **D.** 18.0 m

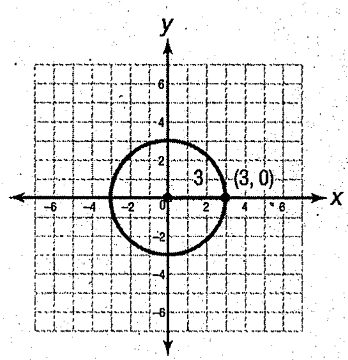
Solve.

28. A scientist uses a satellite in orbit to estimate the diameter of Earth. When the satellite is directly overhead, she sends a signal to the satellite in order to measure its altitude. She records a distance above her of 1,000 miles. As the Earth turns, the satellite eventually moves to the horizon. At this time, the scientist sends another signal and calculates that the satellite is 3,000 miles from her. What is the approximate diameter of the Earth, based on these measures?



____

29. **EXPLAIN** Dilate the circle below twice. One dilation should produce an enlargement. The other should produce a reduction. Identify the scale factor you used for each. Are all three circles similar to one another? Explain why this is so.

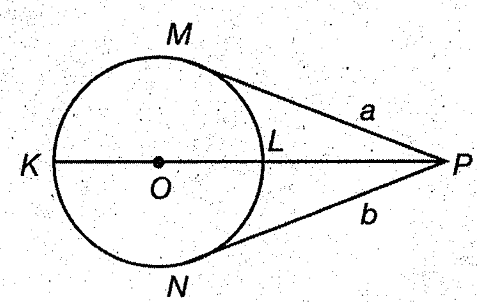


____

____

____

30. **PROVE** Marcus made this statement: "Two tangent segments drawn from the same point *P* outside of a circle must be congruent." Is he correct? Use Circle *O*, tangent segments *PM* and *PN*, and secant segment *PK* to justify your answer.



____

____

____

____