LESSON 30

Set Theory

Sets, Subsets, and Sample Spaces

UNDERSTAND A **set** is a collection of elements, such as objects or numbers. Imagine that you have a bag containing four cards with a number printed on each card. The numbers on the cards are listed below.

4 7 8 9

You can indicate a set by listing its elements in braces. For example, the set of numbers on the cards is {4, 7, 8, 9}, which we can refer to as set *A*. So, $A = \{4, 7, 8, 9\}$.

If every element in a set also belongs to another set, then the first set is a **subset** of the second set. For example, if set $B = \{4, 7\}$, then set *B* is a subset of set *A* because every element of set *B* is also an element of set *A*.

A set containing all possible elements is called the **universal set**, or parent set, denoted by the letter *U*. For sets *A* and *B*, the universal set might be the digits, 0–9. A set containing no elements is called the empty set, or null set, and is indicated by the symbol $\varnothing$. The empty set is a subset of every set.

A set can have a **complement**, which includes all of the elements in the universal set that are not included in that set. The complement of subset *B* is denoted as $\overline{B}$ or B'.

$\overline{B} = \{0, 1, 2, 3, 5, 6, 8, 9\}$

UNDERSTAND Sets can have different relationships. Consider the following sets:

$C = \{7, 8\}$ $D = \{7, 11\}$

The **intersection** of sets, shown by the symbol $\cap$, consists only of the elements that the two sets have in common. You can think of the symbol $\cap$ as meaning "and." The intersection of *C* and *D* contains elements that are in set *C* and in set *D*.

$C \cap D = \{7\}$

The **union** of sets, shown by the symbol $\cup$, consists of all the elements contained in either or both sets. You can think of the symbol $\cup$ as meaning "or." The union of *C* and *D* contains elements that are either in set *C* or in set *D* or in both sets.

$C \cup D = \{7, 8, 11\}$

UNDERSTAND You can use what you know about sets to understand and describe **probabilities**. The **sample space** for a probability experiment is the set of all the possible **outcomes** for the experiment. So, the sample space for tossing a standard number cube is {1, 2, 3, 4, 5, 6}. An **event** in a probability experiment is a subset of the sample space. When a number cube is tossed, you can define any number of events, such as tossing a 2—in which case, the subset is {2}—or tossing an even number—in which case, the subset is {2, 4, 6}.

Venn Diagrams

UNDERSTAND Suppose a universal set consists of all the single-digit whole numbers.

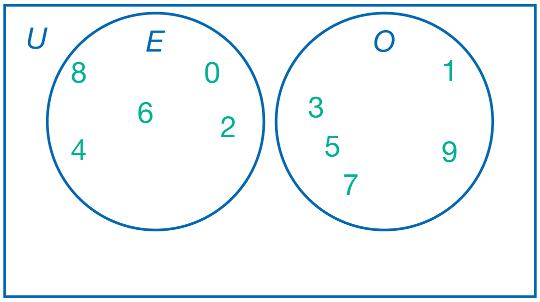
$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Many subsets can be selected from this set. For example, the set of all even numbers, set E, and the set of all odd numbers, O, would be:

$E = \{0, 2, 4, 6, 8\}$ $\quad O = \{1, 3, 5, 7, 9\}$

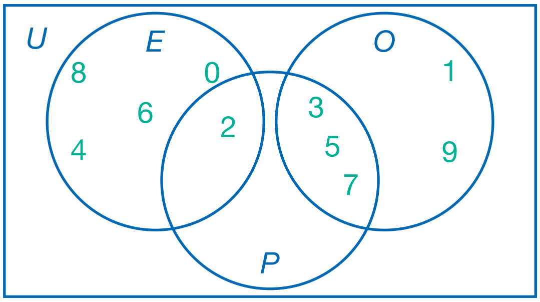
Another way to represent these sets would be to use a Venn diagram. In the Venn diagram below, the large rectangle shows every item in the universal set, U. Subsets E and O are represented as circles inside the larger rectangle. The circles do not overlap. This means that while all of the numbers are part of the universal set, none of the numbers are common to both subsets. From observing the diagram, you can see the following:

- $E \cup O = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = U$
- $E \cap O = \varnothing$
- $\overline{E} = \{1, 3, 5, 7, 9\} = O$
- $\overline{O} = \{0, 2, 4, 6, 8\} = E$



Consider the subset of single-digit whole numbers that are also prime numbers. This set could be represented as $P = \{2, 3, 5, 7\}$. If a circle for set P is added to the Venn diagram, it will overlap with the circle for E (since 2 is prime) and with the circle for O (since 3, 5, and 7 are prime). This diagram allows us to visualize the following

- $E \cap P = \{2\}$
- $O \cap P = \{3, 5, 7\}$
- $\overline{P} = \{0, 1, 4, 6, 8, 9\}$
- $E - P = E \cap \overline{P} = \{0, 4, 6, 8\}$



As shown above, there is often more than one way to describe data by using set notation. For example, to represent the values that are in set E but not in set P, a set difference, $E - P$, or an intersection with a complement, $E \cap \overline{P}$, could be used.

It is important to consider what Venn diagrams do and do not show. For example, $\overline{P}$, the complement of set P, shows the single-digit numbers that are not prime, but that does not mean that it shows the composite numbers. (0 and 1 are neither prime nor composite.) However, it can be used to show that 2 is the only even prime number, especially if additional whole numbers are added.

Practice

Using the information below for questions 1–3.

$U = \{1, 3, 5, 6, 8, 10\}$ $A = \{3, 5, 10\}$ $B = \{3, 5, 6, 8\}$

1. Find $A \cup B$. ____________

2. Find $A \cap B$. ____________

3. Find $\overline{B}$. ____________

REMEMBER $\overline{B}$ means the complement of B.

Write the sample space for each situation.

4. the result of tossing a fair coin ____________

5. the suit of a card chosen from a deck of standard playing cards

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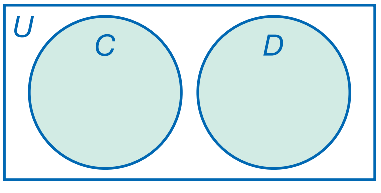
6. the coin you receive as change for a cash purchase ____________

REMEMBER A sample space is the set of all possible outcomes.

Use set notation (e.g., ∪, ∩, ~) to describe the shaded portion of each Venn diagram.

7.



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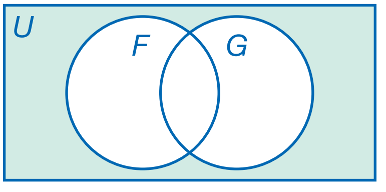

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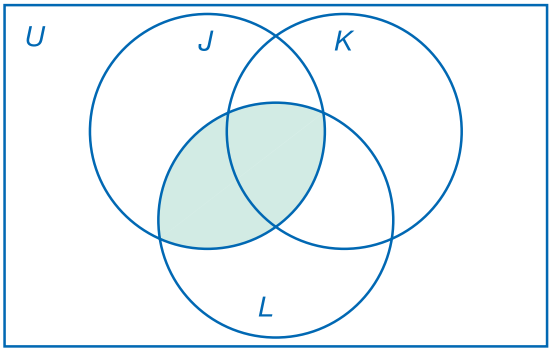

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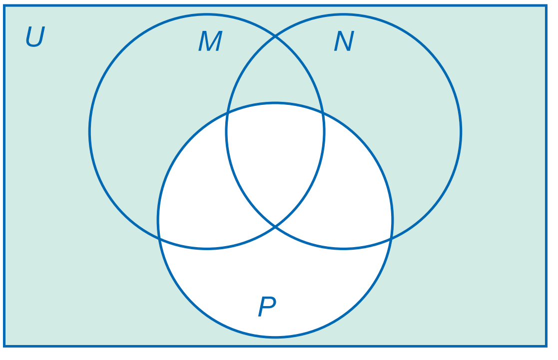

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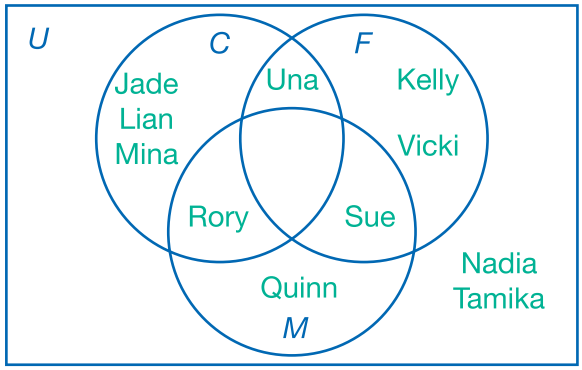
The Venn diagram shows the arts electives that members of a girls' swim team are taking this year in school: Creative Writing (*C*), Fine Arts (*F*), and Music Appreciation (*M*). Let the entire diagram represent *U*, the universal set. Use the diagram for questions 11–13.



11. Find $C \cup F$. What does it represent?

12. Find $C \cap F$. What does it represent?

13. Find $\overline{M \cup C}$. What does it represent?

14. DRAW Lorenzo and Maggie are spinning a spinner with 8 equal-size sections, numbered 1 to 8. If the spinner lands on a composite number (a number with more than 2 factors), Lorenzo gets a point. If the spinner lands on a number less than 4, Maggie gets a point. Draw a Venn diagram to show set *L*, the winning outcomes for Lorenzo, and set *M*, the winning outcomes for Maggie.

Do sets *L* and *M* intersect? Explain why.

15. CREATE A universal set consisting of elements that are plane figures is divided into two subsets, *A* and *B*. $A \cup B$ = {circle, pentagon, rectangle, triangle}, and $A \cap B$ = {triangle}. Create three possible sets, *U*, *A*, and *B*, that fit the description. Are the sets you chose the only possible sets? Explain.