

1.3 Representing and Describing Transformations

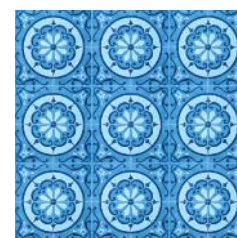


Resource Locker

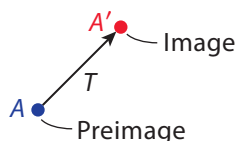
Essential Question: How can you describe transformations in the coordinate plane using algebraic representations and using words?

Explore Performing Transformations Using Coordinate Notation

A **transformation** is a function that changes the position, shape, and/or size of a figure. The inputs of the function are points in the plane; the outputs are other points in the plane. A figure that is used as the input of a transformation is the **preimage**. The output is the **image**. Translations, reflections, and rotations are three types of transformations. The decorative tiles shown illustrate all three types of transformations.



You can use *prime notation* to name the image of a point. In the diagram, the transformation T moves point A to point A' (read “A prime”). You can use function notation to write $T(A) = A'$. Note that a transformation is sometimes called a *mapping*. Transformation T maps A to A' .

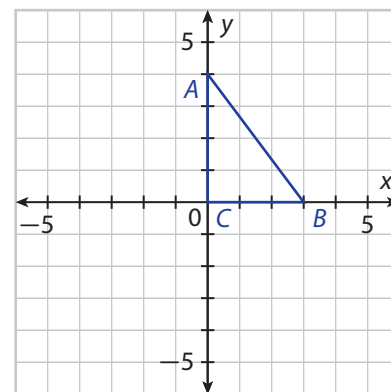


Coordinate notation is one way to write a rule for a transformation on a coordinate plane. The notation uses an arrow to show how the transformation changes the coordinates of a general point, (x, y) .

Find the unknown coordinates for each transformation and draw the image. Then complete the description of the transformation and compare the image to its preimage.

A $(x, y) \rightarrow (x - 4, y - 3)$

Preimage (x, y)	Rule $(x, y) \rightarrow (x - 4, y - 3)$	Image $(x - 4, y - 3)$
$A(0, 4)$	$\rightarrow A'(0 - 4, 4 - 3) =$	$A'(-4, 1)$
$B(3, 0)$	$\rightarrow B'(3 - 4, 0 - 3) =$	$B'(\boxed{?}, \boxed{?})$
$C(0, 0)$	$\rightarrow C'(0 - 4, 0 - 3) =$	$C'(\boxed{?}, \boxed{?})$

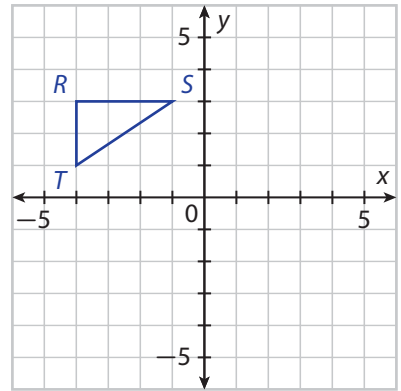


The transformation is a translation 4 units and 3 .

A comparison of the image to its preimage shows that .

B $(x, y) \rightarrow (-x, y)$

Preimage (x, y)	Rule $(x, y) \rightarrow (-x, y)$	Image $(-x, y)$
$R(-4, 3)$	$\rightarrow R'(-(-4), 3)$	$= R'(\boxed{?}, \boxed{?})$
$S(-1, 3)$	$\rightarrow S'(-(-1), 3)$	$= S'(\boxed{?}, \boxed{?})$
$T(-4, 1)$	$\rightarrow T'(-(-4), 1)$	$= T'(\boxed{?}, \boxed{?})$

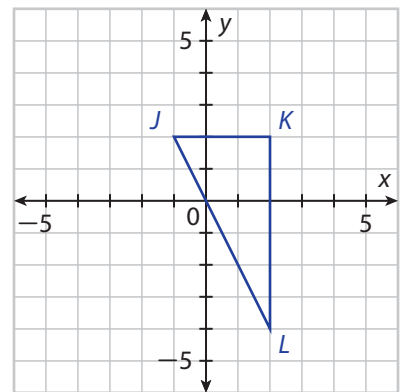


The transformation is a reflection across the $\boxed{?}$ axis.

A comparison of the image to its preimage shows that $\boxed{?}$

C $(x, y) \rightarrow (2x, y)$

Preimage (x, y)	Rule $(x, y) \rightarrow (2x, y)$	Image $(2x, y)$
$J(\boxed{?}, \boxed{?})$	$\rightarrow J'(2 \cdot \boxed{?}, \boxed{?})$	$= J'(\boxed{?}, \boxed{?})$
$K(\boxed{?}, \boxed{?})$	$\rightarrow K'(2 \cdot \boxed{?}, \boxed{?})$	$= K'(\boxed{?}, \boxed{?})$
$L(\boxed{?}, \boxed{?})$	$\rightarrow L'(2 \cdot \boxed{?}, \boxed{?})$	$= L'(\boxed{?}, \boxed{?})$



The transformation is a (horizontal/vertical) stretch by a factor of $\boxed{?}$.

A comparison of the image to its preimage shows that $\boxed{?}$

Reflect

- Discussion** How are the transformations in Steps A and B different from the transformation in Step C?
- For each transformation, what rule could you use to map the image back to the preimage?

Explain 1 Describing Rigid Motions Using Coordinate Notation

Some transformations preserve length and angle measure, and some do not. A **rigid motion** (or *isometry*) is a transformation that changes the position of a figure without changing the size or shape of the figure. Translations, reflections, and rotations are rigid motions.

Properties of Rigid Motions

- | | |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> • Rigid motions preserve distance. • Rigid motions preserve angle measure. • Rigid motions preserve betweenness. | <ul style="list-style-type: none"> • Rigid motions preserve collinearity. • Rigid motions preserve parallelism. |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|

If a figure is determined by certain points, then its image after a rigid motion is determined by the images of those points. This is true because of the betweenness and collinearity properties of rigid motions. Rotations and translations also preserve *orientation*. This means that the order of the vertices of the preimage and image are the same, either clockwise or counterclockwise. Reflections do not preserve orientation.

Example 1 Use coordinate notation to write the rule that maps each preimage to its image. Then identify the transformation and confirm that it preserves length and angle measure.

A Preimage	→	Image
$A(1, 2)$	→	$A'(-2, 1)$
$B(4, 2)$	→	$B'(-2, 4)$
$C(3, -2)$	→	$C'(2, 3)$

Look for a pattern in the coordinates.

The x -coordinate of each image point is the opposite of the y -coordinate of its preimage.

The y -coordinate of each image point equals the x -coordinate of its preimage.

The transformation is a rotation of 90° counterclockwise around the origin given by the rule $(x, y) \rightarrow (-y, x)$.

Find the length of each side of $\triangle ABC$ and $\triangle A'B'C'$. Use the Distance Formula as needed.

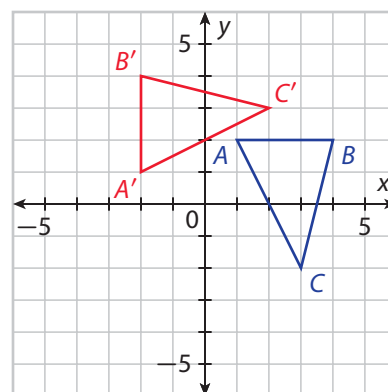
$AB = 3$	$A'B' = 3$
$BC = \sqrt{(3-4)^2 + (-2-2)^2}$	$B'C' = \sqrt{(2-(-2))^2 + (3-4)^2}$
$= \sqrt{17}$	$= \sqrt{17}$
$AC = \sqrt{(3-1)^2 + (-2-2)^2}$	$A'C' = \sqrt{(2-(-2))^2 + (3-1)^2}$
$= \sqrt{20}$	$= \sqrt{20}$

Since $AB = A'B'$, $BC = B'C'$, and $AC = A'C'$, the transformation preserves length.

Find the measure of each angle of $\triangle ABC$ and $\triangle A'B'C'$. Use a protractor.

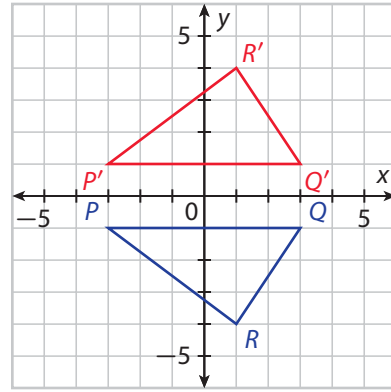
$$m\angle A = 63^\circ, m\angle B = 76^\circ, m\angle C = 41^\circ \quad m\angle A' = 63^\circ, m\angle B' = 76^\circ, m\angle C' = 41^\circ$$

Since $m\angle A = m\angle A'$, $m\angle B = m\angle B'$, and $m\angle C = m\angle C'$, the transformation preserves angle measure.



B

Preimage	Image
$P(-3, -1) \rightarrow$	$P'(-3, 1)$
$Q(3, -1) \rightarrow$	$Q'(3, 1)$
$R(1, -4) \rightarrow$	$R'(1, 4)$



Look for a pattern in the coordinates.

The x -coordinate of each image point equals the x -coordinate of its preimage.

The y -coordinate of each image point is the opposite of the y -coordinate of its preimage.

The transformation is a reflection across the x -axis given by the rule $(x, y) \rightarrow (x, -y)$.

Find the length of each side of $\triangle PQR$ and $\triangle P'Q'R'$.

$$PQ = \boxed{6}$$

$$P'Q' = \boxed{6}$$

$$\begin{aligned} QR &= \sqrt{(1 - 3)^2 + (-4 - (-1))^2} \\ &= \sqrt{\boxed{13}} \end{aligned}$$

$$\begin{aligned} Q'R' &= \sqrt{(1 - 3)^2 + (4 - 1)^2} \\ &= \sqrt{\boxed{13}} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(1 - (-3))^2 + (-4 - (-1))^2} \\ &= \sqrt{\boxed{25}} = \boxed{5} \end{aligned}$$

$$\begin{aligned} P'R' &= \sqrt{(1 - (-3))^2 + (4 - (-1))^2} \\ &= \sqrt{\boxed{25}} = \boxed{5} \end{aligned}$$

Since $PQ = P'Q'$, $QR = Q'R'$, and $PR = P'R'$, the transformation preserves length.

Find the measure of each angle of $\triangle PQR$ and $\triangle P'Q'R'$. Use a protractor.

$$m\angle P = \boxed{37^\circ}, m\angle Q = \boxed{56^\circ}, m\angle R = \boxed{87^\circ} \quad m\angle P' = \boxed{37^\circ}, m\angle Q' = \boxed{56^\circ}, m\angle R' = \boxed{87^\circ}$$

Since $m\angle P = m\angle P'$, $m\angle Q = m\angle Q'$, and $m\angle R = m\angle R'$, the transformation preserves angle measure.

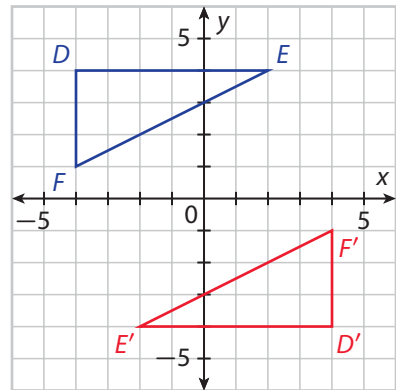
Reflect

- How could you use a compass to test whether corresponding lengths in a preimage and image are the same?
- Look back at the transformations in the Explore. Classify each transformation as a rigid motion or not a rigid motion.

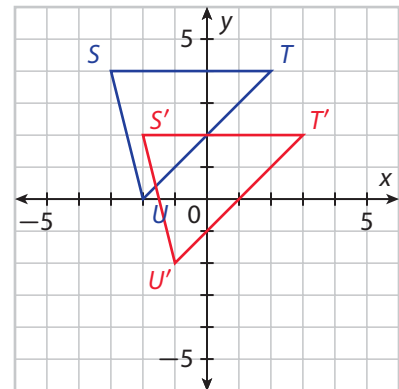
Your Turn

Use coordinate notation to write the rule that maps each preimage to its image. Then identify the transformation and confirm that it preserves length and angle measure.

- 5. Preimage** **Image**
 $D(-4, 4)$ \rightarrow $D'(4, -4)$
 $E(2, 4)$ \rightarrow $E'(-2, -4)$
 $F(-4, 1)$ \rightarrow $F'(4, -1)$



- 6. Preimage** **Image**
 $S(-3, 4)$ \rightarrow $S'(-2, 2)$
 $T(2, 4)$ \rightarrow $T'(3, 2)$
 $U(-2, 0)$ \rightarrow $U'(-1, -2)$



Explain 2 **Describing Nonrigid Motions Using Coordinate Notation**

Transformations that stretch or compress figures are not rigid motions because they do not preserve distance.

The view in the fun house mirror is an example of a vertical stretch.



© Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Mary Hockenbery/Flickr/Getty Images

Example 2 Use coordinate notation to write the rule that maps each preimage to its image. Then confirm that the transformation is not a rigid motion.

A $\triangle JKL$ maps to triangle $\triangle J'K'L'$.

Preimage	Image
$J(4, 1)$	$\rightarrow J'(4, 3)$
$K(-2, -1)$	$\rightarrow K'(-2, -3)$
$L(0, -3)$	$\rightarrow L'(0, -9)$

Look for a pattern in the coordinates.

The x -coordinate of each image point equals the x -coordinate of its preimage.
 The y -coordinate of each image point is 3 times the y -coordinate of its preimage.
 The transformation is given by the rule $(x, y) \rightarrow (x, 3y)$.

Compare the length of a segment of the preimage to the length of the corresponding segment of the image.

$$\begin{aligned}
 JK &= \sqrt{(-2 - 4)^2 + (-1 - 1)^2} & J'K' &= \sqrt{(-2 - 4)^2 + (-3 - 3)^2} \\
 &= \sqrt{40} & &= \sqrt{72}
 \end{aligned}$$

Since $JK \neq J'K'$, the transformation is not a rigid motion.

B $\triangle MNP$ maps to triangle $\triangle M'N'P'$.

Preimage	Image
$M(-2, 2)$	$\rightarrow M'(-4, 1)$
$N(4, 0)$	$\rightarrow N'(8, 0)$
$P(-2, -2)$	$\rightarrow P'(-4, -1)$

The x -coordinate of each image point is twice the x -coordinate of its preimage.

The y -coordinate of each image point is half the y -coordinate of its preimage.

The transformation is given by the rule $(x, y) \rightarrow (2x, \frac{1}{2}y)$.

Compare the length of a segment of the preimage to the length of the corresponding segment of the image.

$$\begin{aligned}
 MN &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} & M'N' &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(4 - (-2))^2 + (0 - 2)^2} & &= \sqrt{(8 - (-4))^2 + (0 - 1)^2} \\
 &= \sqrt{6^2 + (-2)^2} & &= \sqrt{12^2 + (-1)^2} \\
 &= \sqrt{40} & &= \sqrt{145}
 \end{aligned}$$

Since $MN \neq M'N'$, the transformation is not a rigid motion.

Reflect

7. How could you confirm that a transformation is not a rigid motion by using a protractor?

Your Turn

Use coordinate notation to write the rule that maps each preimage to its image. Then confirm that the transformation is not a rigid motion.

8. $\triangle ABC$ maps to triangle $\triangle A'B'C'$.

Preimage	Image
$A(2, 2)$	$\rightarrow A'(3, 3)$
$B(4, 2)$	$\rightarrow B'(6, 3)$
$C(2, -4)$	$\rightarrow C'(3, -6)$

9. $\triangle RST$ maps to triangle $\triangle R'S'T'$.

Preimage	Image
$R(-2, 1)$	$\rightarrow R'(-1, 3)$
$S(4, 2)$	$\rightarrow S'(2, 6)$
$T(2, -2)$	$\rightarrow T'(1, -6)$

Elaborate

10. **Critical Thinking** To confirm that a transformation is not a rigid motion, do you have to check the length of every segment of the preimage and the length of every segment of the image? Why or why not?
11. **Make a Conjecture** A polygon is transformed by a rigid motion. How are the perimeters of the preimage polygon and the image polygon related? Explain.
12. **Essential Question Check-In** How is coordinate notation for a transformation, such as $(x, y) \rightarrow (x + 1, y - 1)$, similar to and different from algebraic function notation, such as $f(x) = 2x + 1$?

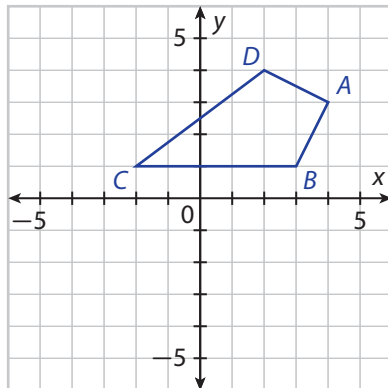
Evaluate: Homework and Practice



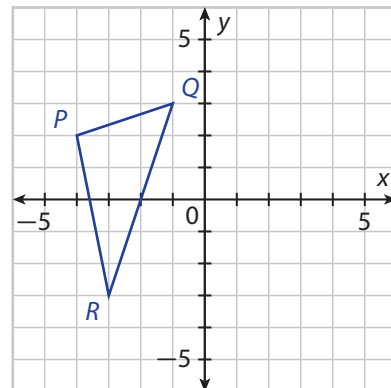
- Online Homework
- Hints and Help
- Extra Practice

On a coordinate grid, draw each figure and its image under the given transformation. Then describe the transformation in words.

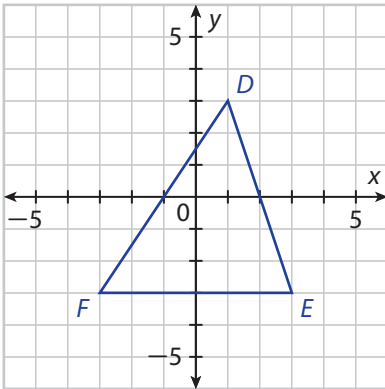
1. $(x, y) \rightarrow (-x, -y)$



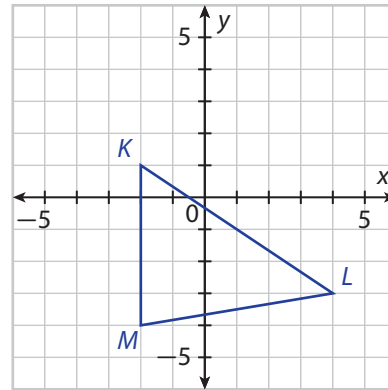
2. $(x, y) \rightarrow (x + 5, y)$



3. $(x, y) \rightarrow (x, \frac{1}{3}y)$



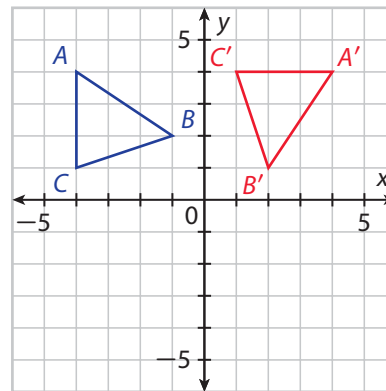
4. $(x, y) \rightarrow (y, x)$



Use coordinate notation to write the rule that maps each preimage to its image. Then identify the transformation and confirm that it preserves length and angle measure.

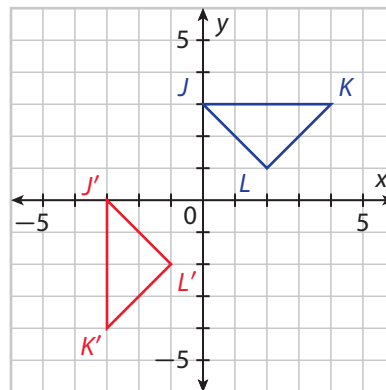
5. Preimage Image

$$\begin{aligned} A(-4, 4) &\rightarrow A'(4, 4) \\ B(-1, 2) &\rightarrow B'(2, 1) \\ C(-4, 1) &\rightarrow C'(1, 4) \end{aligned}$$



6. Preimage Image

$$\begin{aligned} J(0, 3) &\rightarrow J'(-3, 0) \\ K(4, 3) &\rightarrow K'(-3, -4) \\ L(2, 1) &\rightarrow L'(-1, -2) \end{aligned}$$



Use coordinate notation to write the rule that maps each preimage to its image. Then confirm that the transformation is not a rigid motion.

7. $\triangle ABC$ maps to triangle $\triangle A'B'C'$.

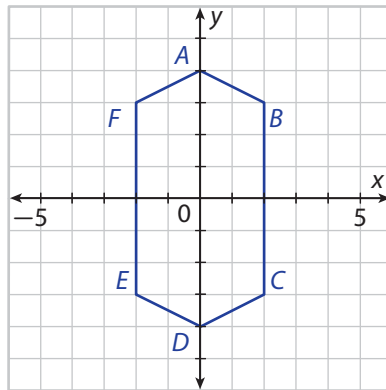
Preimage	Image
$A(6, 6) \rightarrow$	$A'(3, 3)$
$B(4, -2) \rightarrow$	$B'(2, -1)$
$C(0, 0) \rightarrow$	$C'(0, 0)$

8. $\triangle FGH$ maps to triangle $\triangle F'G'H'$.

Preimage	Image
$F(-1, 1) \rightarrow$	$F'(-2, 1)$
$G(1, -1) \rightarrow$	$G'(2, -1)$
$H(-2, -2) \rightarrow$	$H'(-4, -2)$



Analyze Relationships A mineralogist is studying a quartz crystal. She uses a computer program to draw a side view of the crystal, as shown. She decides to make the drawing 50% wider, but to keep the same height. Draw the transformed view of the crystal. Then write a rule for the transformation using coordinate notation. Check your rule using the original coordinates.



10. Use the points $A(2, 3)$ and $B(2, -3)$.

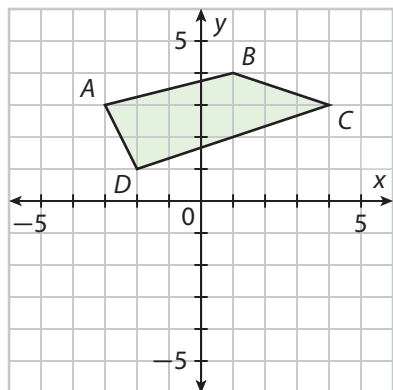
- Describe segment AB and find its length.
- Describe the image of segment AB under the transformation $(x, y) \rightarrow (x, 2y)$.
- Describe the image of segment AB under the transformation $(x, y) \rightarrow (x + 2, y)$.
- Compare the two transformations.

11. Use the points $H(-4, 1)$ and $K(4, 1)$.

- Describe segment HK and find its length.
- Describe the image of segment HK under the transformation $(x, y) \rightarrow (-y, x)$.
- Describe the image of segment HK under the transformation $(x, y) \rightarrow (2x, y)$.
- Compare the two transformations.



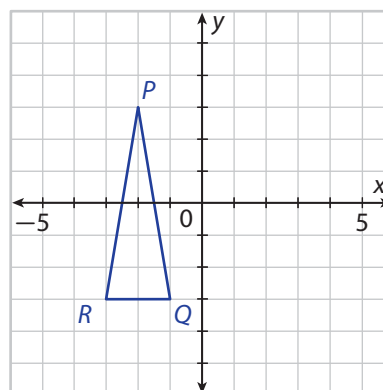
- 12. Make a Prediction** A landscape architect designs a flower bed that is a quadrilateral, as shown in the figure. The plans call for a light to be placed at the midpoint of the longest side of the flower bed. The architect decides to change the location of the flower bed using the transformation $(x, y) \rightarrow (x, -y)$. Describe the location of the light in the transformed flower bed. Then make the required calculations to show that your prediction is correct.



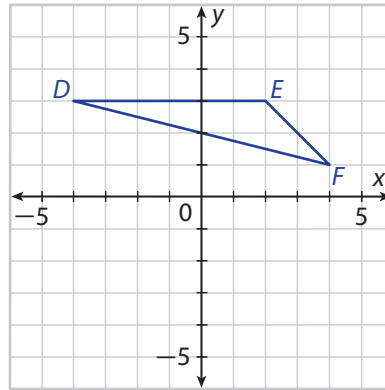
- 13. Multiple Representations** If a transformation moves points only up or down, how do the coordinates of the point change? What can you conclude about the coordinate notation for the transformation?
- 14.** Match each transformation with the correct description.
- | | |
|------------------------------------|------------------------------------------------|
| A. $(x, y) \rightarrow (3x, y)$ | a. ___?___ dilation with scale factor 3 |
| B. $(x, y) \rightarrow (x + 3, y)$ | b. ___?___ translation 3 units up |
| C. $(x, y) \rightarrow (x, 3y)$ | c. ___?___ translation 3 units right |
| D. $(x, y) \rightarrow (x, y + 3)$ | d. ___?___ horizontal stretch by a factor of 3 |
| E. $(x, y) \rightarrow (3x, 3y)$ | e. ___?___ vertical stretch by a factor of 3 |

On a coordinate grid, draw each figure and its image under the given transformation. Then describe the transformation as a rigid motion or not a rigid motion. Justify your answer.

- 15.** $(x, y) \rightarrow (2x + 4, y)$



16. $(x, y) \rightarrow (0.5x, y - 4)$

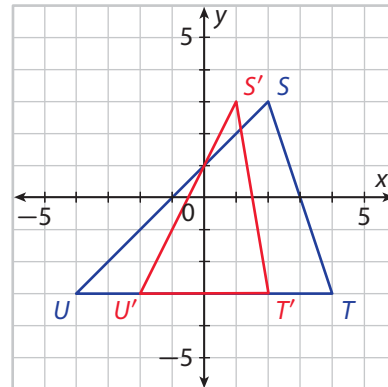


H.O.T. Focus on Higher Order Thinking

17. **Explain the Error** A student claimed that the transformation $(x, y) \rightarrow (3x, y)$ is a rigid motion because the segment joining $(5, 0)$ to $(5, 2)$ is transformed to the segment joining $(15, 0)$ to $(15, 2)$, and both of these segments have the same length. Explain the student's error.

18. **Critical Thinking** Write a rule for a transformation that maps $\triangle STU$ to $\triangle S'T'U'$.

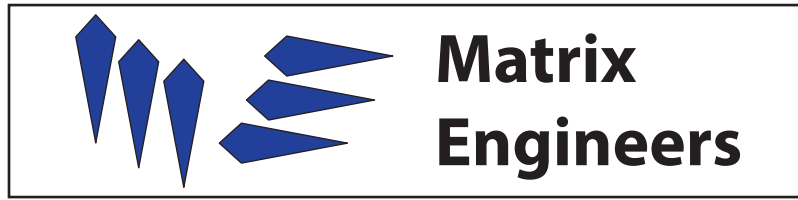
19. **Justify Reasoning** Consider the transformation given by the rule $(x, y) \rightarrow (0, 0)$. Describe the transformation in words. Then explain whether or not the transformation is a rigid motion and justify your reasoning.



20. **Communicate Mathematical Ideas** One of the properties of rigid motions states that rigid motions preserve parallelism. Explain what this means, and give an example using a specific figure and a specific rigid motion. Include a graph of the preimage and image.

Lesson Performance Task

A Web designer has created the logo shown here for Matrix Engineers.



The logo is 100 pixels wide and 24 pixels high. Images placed in Web pages can be stretched horizontally and vertically by changing the dimensions in the code for the Web page.

The Web designer would like to change the dimensions of the logo so that lengths are increased or decreased but angle measures are preserved.

- a. Find three different possible sets of dimensions for the width and height so that lengths are changed but angle measures are preserved. The dimensions must be whole numbers of pixels. Justify your choices.
- b. Explain how the Web designer can use transformations to find additional possible dimensions for the logo.